# Abstracts

### Quantitative results in stochastic homogenization for divergence-form equations.

*Armstrong, Scott (Paris-Dauphine)*

I will summarize some recent progress in the study of linear, divergence form elliptic equations with random coefficients. Several years ago, Gloria and Otto and collaborators used an idea from statistical mechanics (concentration of measure) to establish strong quantitative bounds on the sizes of fluctuations of solutions, and then Nolen, Mourrat and others proved central limit theorems for Dirichlet forms and obtained the scaling limits of solutions. In all of these results, concentration inequalities were the essential tool for transferring ergodic information from the coefficients (which satisfy an iid assumption) to the solutions themselves, and this requires some restrictive assumptions. In this talk, I will describe an alternative approach developed in the last several years with Smart and more recently with Kuusi and Mourrat. We obtain optimal quantitative estimates, central limit theorems, and the scaling limit of the correctors to a variant of the Gaussian Free Field-- under fewer assumptions, with much stronger stochastic integrability, and without using abstract concentration of measure. Rather, our approach is to "linearize the randomness around the homogenized limit": that is, we use renormalization arguments which reveal that certain Dirichlet forms are essentially additive quantities. The complicated nonlinear structure of the randomness is reduced to a linear one (essentially a sum of iid random variables) and a complete quantitative theory can then be easily read off.

### Liouville properties of fully nonlinear possibly degenerate elliptic operators and some applications.

*Bardi, Martino (Padova)*

I will present a joint paper with Annalisa Cesaroni (Padova). We prove some Liouville properties for sub- and supersolutions of fully nonlinear degenerate elliptic equations in the whole space. Our assumptions allow the coefficients of the first order terms to be large at infinity, provided they have an appropriate sign, as in Ornstein-Uhlenbeck operators. We give two applications. The first is a stabilization property for large times of solutions to fully nonlinear parabolic equations. The second is the solvability of an ergodic Hamilton-Jacobi-Bellman equation that identifies a unique critical value of the operator.

### Surprise and predictability in bounded sources of non-convex balance laws.

*Caravenna, Laura (Padova)*

In the talk I will show surprising and predictable aspects of bounded source terms in a non-convex balance law, with smooth flux, when it admits a continuous solution. Namely, I will discuss to what extent the conservation law can be reduced to an (infinitely dimensional) system of ODEs along the characteristic curves. This correspondence is evident in the classical setting but it is surprising in this context with lack of regularity. Part of the correspondence just requires suitable definitions and smart technicality, but concerning part of it new odd unexpected behaviors show up. The presentation is mostly based on a joint work with S. Bianchini (SISSA) and G. Alberti (Pisa), and it extends previous works by several authors relative to the case of the quadratic flux.

### The master equation and the convergence problem in Mean Field Games.

*Cardaliaguet, Pierre (Paris-Dauphine)*

We will discuss the convergence, as N tends to infinity, of a system of N coupled Hamilton-Jacobi equations, the Nash system. This system arises in differential game theory. We describe the limit problem in terms of the so-called "master equation", a kind of transport equation stated on the space of probability measures. This is a just work with F. Delarue, J.-M. Lasry and P.L. Lions.

### Homogenization of a semilinear heat equation.

*Cesaroni, Annalisa (Padova)*

I will discuss the homogenization of a semilinear heat equation with vanishing viscosity and oscillating potential depending on u/eps. According to the rate between frequency of oscillations and vanishing factor in the viscosity, we obtain different limit behaviour of the solutions. In the weak diffusion regime, the effective operator is discontinuous in the gradient entry, an unusual phenomenon in homogenization, and makes the analysis of the limit more challenging.
Joint work with Dirr (Cardiff) and Novaga (Pisa).

### Multidimensional Shock waves and Free boundary problems for Nonlinear PDEs of Mixed Type.

*Gui-Qiang G. Chen (Oxford)*

In this talk, we will discuss some recent progress in the analysis of multidimensional shock waves and related free boundary problems for nonlinear PDEs of mixed elliptic-hyperbolic type.
Further trends and open problems in this direction and their connections with some fundamental problems in other areas will also be addressed if time permits.

### Everywhere discontinuous anisotropy of thin periodic composite plates.

*Mikhail Cherdantsev (Cardiff)*

We consider an elastic periodic composite plate in the full bending regime, i.e. when the displacement of the plate is of finite order. Both the thickness of the plate h and the period of the composite structure ε are small parameters. We start from the non-linear elasticity setting. Passing to the limit as h, ε→, 0 we carry out simultaneous dimension reduction and homogenisation to obtain an effective limit elastic functional which describes the asymptotic properties of the composite plate. We show, in particular, that in the regime $h \ll \varepsilon^2$ the limit elastic functional is discontinuously anisotropic in every direction of bending. This remarkable property (suggesting that the corresponding composite plate can be referred to as metamaterial) is due to the in-limit linearisation of the bending deformations and the multi scale interaction.

## Homogenisation in finite elasticity for composites with a high contrast in the vicinity of rigid-body motions

*Cherednichenko, Kirill (Bath)*

I will describe a multiscale asymptotic framework for the analysis of the macroscopic behaviour of periodic two-material composites with high contrast in a finite-strain setting. I will start by introducing the nonlinear description of a composite consisting of a stiff material matrix and soft, periodically distributed inclusions. I shall then focus on the loading regimes when the applied load is small or of order one in terms of the period of the composite structure. I will show that this corresponds to the situation when the displacements on the stiff component are situated in the vicinity of a rigid-body motion. This allows to replace, in the homogenisation limit, the nonlinear material law of the stiff component by its linearised version. As a main result, I derive (rigorously in the spirit of Γ-convergence) a limit functional that allows to establish a precise two-scale expansion for minimising sequences. This is joint work with Mikhail Cherdantsev and Stefan Neukamm.

## Compensated convexity, multiscale medial axis maps, and sharp regularity of the squared distance function

*Crooks, Elaine (Swansea)*

Compensated convex transforms enjoy tight-approximation and locality properties that can be exploited to develop multiscale, parametrised methods for identifying singularities in functions. When applied to the squared distance function to a closed subset of Euclidean space, these ideas yield a new tool for locating and analyzing the medial axis of geometric objects, called the multiscale medial axis map. This consists of a parametrised family of nonnegative functions that provides a Hausdorff-stable multiscale representation of the medial axis, in particular producing a hierarchy of heights between different parts of the medial axis depending on the distance between the generating points of that part of the medial axis. Such a hierarchy enables subsets of the medial axis to be selected by simple thresholding, which tackles the well-known stability issue that small perturbations in an object can produce large variations in the corresponding medial axis. A sharp regularity result for the squared distance function is obtained as a by-product of the analysis of this multiscale medial axis map.
This is joint work with Kewei Zhang, Nottingham, and Antonio Orlando, Tucumán.

## A phase field model for Willmore's energy with topological constraint

*Dondl, Patrick (Freiburg)*

We consider the problem of minimizing Willmore's energy on confined and connected surfaces with prescribed surface area. To this end, we approximate the surface by a level set function u admitting the value +1 on the inside of the surface and -1 on its outside. The confinement of the surface is now simply given by the domain of definition of u. A diffuse interface approximation for the area functional, as well as for Willmore's energy are well known. We address the main difficulty, namely the topological constraint of connectedness by a penalization of a geodesic distance which is chosen to be sensitive to connected components of the phase field level sets and provide a proof of Gamma-convergence of our model to the sharp interface limit. Furthermore, we show some numerical results.
This is joint work with Stephan Wojtowytsch (Durham University) and Antoine Lemenant (University Paris 7).

## Pathwise Well-Posedness of the Fast Diffusion Equation with Affine Multiplicative Noise

*Fehrman, Benjamin (MPI-MIS Leipzig)*

In this talk, which describes joint work with Benjamin Gess, I will discuss the existence and uniqueness of pathwise entropy solutions for the fast diffusion equation driven by affine multiplicative noise. The theory of such solutions is motivated by the study of stochastic viscosity solutions, and was first developed in the context of scalar conservation laws with rough fluxes by Lions, Perthame and Souganidis, and later extended by Gess and Souganidis. Their approach is based upon the kinetic formulation of the equation, and involves testing the solution against data propagating along the corresponding path-dependent characteristics. I hope to describe the analogous theory in the context of the fast-diffusion equation, and to explain how it can be used to establish the well-posedness of pathwise entropy solutions in this setting.

## Some results on stochastic homogenization of non-convex Hamilton-Jacobi equations

*Feldman, William (Chicago)*

I will discuss some progress about the homogenization of non-convex Hamilton-Jacobi equations in random media. I will revisit the recent counter-example of Ziliotto who constructed a coercive but non-convex Hamilton-Jacobi equation with stationary ergodic random potential field for which homogenization does not hold. We have extended this result showing that for any Hamiltonian with a strict saddle-point there is a random stationary ergodic potential field V so that homogenization does not hold for the Hamiltonian H = h(p)-V(x). I will also discuss a positive result, under a finite range of dependence assumption we show that homogenization holds for Hamiltonians with strictly star-shaped sub-level sets.
This talk is based on joint work with P. Souganidis.

## Hölder continuity for solutions of eikonal equations

*Feleqi, Ermal (Cardiff)*

I will talk about results on Hölder continuity of viscosity solutions of eikonal PDEs

$$\begin{cases} |\nabla_X u| = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

structured on possibly degenerate and nonsmooth systems of vector fields $X = (X_1, \dots, X_p)$. The typical result goes as follows: if the given vector fields satisfy Hör- mander's condition of at most step k at each interior point of Ω and if at each boundary point of Ω a Lie bracket of degree at most k can by found not tangential (i.e., transversal) to ∂Ω, then the solution of (P) is (1/k)- Hölder continuous. The proof relies on representing u as the value function of an optimal control problem: actually, when $f \equiv 1$, as the minimum time to reach the exterior of Ω by X -trajectories, that is, concatenations of a finite number of integral curves of the vector fields $\pm X_i$, i = 1, . . . , p. The smoothness assumptions on vector fields and ∂Ω are reduced to a "bare minimum". For vector fields this is made possible by introducing a set-valued notion of iterated Lie bracket which makes sense for quite nonsmooth vector fields. Concerning the regularity of Ω, we require for it to satisfy an exterior cone condition. Then the transversality condition is expressed by requiring that all the vectors of a set-valued bracket point toward an exterior cone. When Ω is of class $C^1$ or possesses Bony normals the transversality condition can be phrased in more natural terms (every vector of the bracket

should not be orthogonal to the normal). We can cover also boundaries with isolated points. At those isolated points the trensversality condition is expressed by requiring that Hörmander's condition be satisfied therein.
Joint work with Franco Rampazzo, Martino Bard and Pierpaolo Soravia

### New monotonicity formulas in free boundary problems and their applications.

*Garofalo, Nicola (Padova)*

In this talk I will overview various new monotonicity formulas in free boundary problems of obstacle type and discuss their applications to questions such as optimal regularity of the solution, uniqueness of blow-ups and regularity of the free boundary.

### On a class of Optimal Transport problems with repulsive cost functions

*Gerolin, Augusto (Bath/Pisa)*

The goal of this communication is to present a new class of optimal transport problems with repulsive cost functions motivated by Quantum Mechanics (computation of the ground state energy of an Electronic Schrödinger Equation). I want highlight some issues and point out our progress regarding the existence of Monge-type solutions in this setting.

### Convergence along mean flows.

*Hutridurga, Harsha (Cambridge)*

This talk shall address the homogenization of parabolic equations of convection-diffusion type with large drift and with locally periodic rapidly oscillating coefficients. We answer an outstanding open problem in the theory of homogenization of parabolic problems. We shall develop a technique of multiple scale asymptotic expansions along mean flows and a corresponding notion of weak multiple scale convergence. Crucial to our analysis is the introduction of a fast time variable. We shall prove that the solution family taken along a particularly chosen rapidly moving coordinate system converges to the solution of a diffusion equation. The effective diffusion coefficient is expressed in terms of the average of Eulerian cell solutions along the orbits of the mean flow in the fast time variable. To make this notion rigorous, we use the theory of ergodic algebras with mean value. This is a joint work with Thomas Holding (Cambridge) and Jeffrey Rauch (Michigan).

### Higher order $L^\infty$ variational problems and the $\infty$-Polylaplacian

*Katzourakis, Nikos (Reading)*

Calculus of Variations in $L^\infty$ is a relatively new field initiated by Aronsson in the 1960s which is under active research since. Minimising the supremum of a function of the gradient is very challenging because the equations arising as the analogues of the Euler-Lagrange equations are non-divergence and highly degenerate. However, it provides more realistic models than the classical average functionals (integrals). In this talk I will discuss a very recent advance made jointly with T. Pryer (Reading, UK), where we initiated the study of 2nd order variational problems in $L^\infty$ , seeking to minimise the $L^\infty$ norm of a function of the hessian. We also derived and studied the associated PDE. The latter is fully nonlinear and of 3rd order. Special cases arise when the function is the Euclidean length of either the full hessian or of the Laplacian, leading to the $\infty$-Polylaplacian and the $\infty$-Bilaplacian respectively. Our analysis relies heavily on the recently proposed by the speaker theory of D-solutions, a general dualityfree notion of generalised solutions for fully nonlinear PDE systems which do not support integration-by-parts.

### Uniaxial versus Biaxial Character of Landau-de Gennes Minimizers in Three Dimensions

*Apala Majumdar (Bath)*

We study global minimizers of the Landau-de Gennes energy functional for nematic liquid crystals, on arbitrary three-dimensional simply connected geometries with topologically non-trivial and physically relevant Dirichlet boundary conditions. Our results are specific to an asymptotic limit defined in terms of a re-scaled reduced temperature, t. We prove (i) that (re-scaled) global LdG minimizers converge uniformly to a (minimizing) limiting harmonic map, away from the singular set of the limiting map; (ii) we have points of maximal biaxiality and uniaxiality near each singular point of the limiting map (this improves recent results of Contreras and Lamy); (iii) estimates for the size of "strongly biaxial" regions in terms of the reduced temperature t. This is joint work with Duvan Henao and Adriano Pisante.

### Mean Field Games on Networks

*Marchi, Claudio (Padova)*

We consider stationary Mean Field Games (briefly, MFG) defined on a network. In this framework, the transition conditions at the vertices play a crucial role: the ones here considered are based on the optimal control interpretation of the problem.
First, we prove separately the well-posedness of each of the two equations composing the MFG system. After we prove existence and uniqueness of the solution to the MFG system.
Finally, we propose some numerical methods, proving the well-posedness and the converging of the scheme.
These are joint works with F. Camilli and S. Cacace.

### Generalized junction conditions for degenerate parabolic equations.

*Nguyen, Vinh (Cardiff)*

We study degenerate parabolic equations in multi-domains whose coefficients are discontinuous along interfaces. We observe that the approach proposed by IMBERT and Monneau (2014) for Hamilton-Jacobi equations can be further developed to handle generalized junction conditions (such as the generalized Kirchoff ones) and second order terms. We first prove that generalized junction conditions reduce to flux-limited ones. We then use then vertex test function (Imbert, Monneau -- 2014) to prove a comparison principle. We then determine the vanishing viscosity limit associated with Hamilton-Jacobi equations posed on multi-domains and networks. In the two-domain and convex case, the maximal Ishii solution identified by Barles, Briani and Chasseigne (2012) is selected. Finally, we give a short and simple PDE proof for the large deviation result of Boue, Dupuis and Ellis (2000). Joint work with C. Imbert.

## Models of charged drops.

*Novaga, Matteo (Pisa)*

Electrified liquids are well known to be prone to a variety of interfacial instabilities that result in the onset of apparent interfacial singularities and liquid fragmentation. In the case of electrically conducting liquids, one of the basic models describing the equilibrium interfacial configurations and the onset of instability assumes the liquid to be equipotential and interprets those configurations as local minimizers of the energy consisting of the sum of the surface energy and the electrostatic energy. Surprisingly, this classical geometric variational model is mathematically ill-posed irrespectively of the degree to which the liquid is electrified. Specifically, an isolated spherical droplet is never a local minimizer, no matter how small is the total charge on the droplet, since the energy can always be lowered by a smooth, arbitrarily small distortion of the droplet's surface. This is in sharp contrast with the experimental observations that a critical amount of charge is needed in order to destabilize a spherical droplet. We discuss some possible regularization mechanisms for the considered free boundary problem.

## Scalar conservation laws with nonlinear multiplicative rough signal.

*Panagiotis E. Souganidis (Chicago)*

I will present a recently developed theory for scalar conservation laws with nonlinear multiplicative rough signal dependence. I will describe the difficulties, introduce the notion of pathwise entropy/kinetic solution and its well-posedness. I will also talk about the long time behavior of the solutions as well as some regularization by noise type results

## A tour of Lipschitz truncations.

*Stroffolini, Bianca (Napoli)*

The purpose of the Lipschitz truncation is to regularize a given function by a Lipschitz continuous one by changing it only on a small bad set. It is crucial for the applications that the function is not changed globally, which rules out the possibility of convolutions.
The Lipschitz truncation technique was introduced by Acerbi-Fusco to show lower semicontinuity of certain variational integrals.
Since then this technique has been successfully applied in many different areas: biting lemmas, existence theory and regularity results of non-linear elliptic PDE. It was also successfully applied in the framework of non-Newtonian fluids of power law type and even in the context of numerical analysis.
I will try to present some Lipschitz truncations Lemmas.
As an application, existence/ regularity of solutions of PDEs will be discussed.

## Posters/Short communications:

### Minimal cost for the macroscopic motion of an interface

*Birmpa, Panagiota (Sussex)*

We will discuss the power needed to force a motion of an interface between two different phases of a given ferromagnetic sample with a prescribed speed V. In this model, the interface is the non-homogeneous stationary solution of a non local evolution equation. Considering a stochastic microscopic system of Ising spins with Kac interaction evolving in time according to Glauber dynamics, we assign the cost functional which penalizes deviations from the solutions of the mesoscopic evolution equation by considering the underlying microscopic process. Then, we study the optimal way to displace the interface

### About numerically extracting the metric for entropy-driven diffusive systems

*Embacher, Peter (Cardiff)*

We consider a diffusive system that is microscopically driven by an underlying stochastic process. For some of these systems it has been shown that their macroscopical evolution can be described by a gradient flow along its entropy, as long as the corresponding thermodynamical metric is chosen conveniently. We propose a numerical method to extract this metric from given experimental data. The method was applied to zero-range-processes and proved successfull there.

### Star-shaped and convex sets in the Heisenberg group.

*Filali, Doaa (Cardiff)*

Star-shaped sets play a signifcant role in analysis and PDEs. In particular convex sets are star-shaped with respect to the all interior points. In the Euclidean case there are several equivalent definitions for star-shaped sets. This is not true in more degenerate geometries. For example in the Heisenberg group we generalised several Euclidean definitions for starshapedness and they turned to be not equivalent. For example we looked at the definition of strong star-shaped sets (related to the dilations) and the definition of weak star-shaped sets (related to the horizontal line segments). We construct counterexamples showing that the two definitions are not equivalent. While the first definition is important for some PDE application, the second weaker definition is key for convexity. In fact weak star-shaped with respect to all interior points is equivalent to the horizontal convexity (and all other equivalent notions known in the Heisenberg group). Beside that we study the relation between many different notions of star-shaped sets and their relations with convexity and convex functions. These results have been applied in the study of geometrical properties for level sets of nonlinear subelliptic PDEs.

## Periodic sets in Grushin spaces with applications to homogenisation.

*Jama, Ahmed (Cardiff)*

We will discuss periodic homogenisation for divergence form boundary problems first in the Heisenberg group with the aim of extending the results to Grushin spaces. The periodicity in the Heisenberg group is adapted to the Lie group structure of the space. Grushin spaces lack of any algebraic structure, therefore the first step is to develop a good notion of periodicity. We introduce a notion of periodicity using translations induced by the vector fields associated to the Grushin space and show that a very different behaviour when these translations are applied to balls centred in the origin (or at any point crossing the y-axis) compared to balls that do not intersect the line x=0.

## Stochastic Filtering for Rotating Shallow Water Equations

*Lang, Oana (Imperial)*

The aim of the talk/poster is to present a stochastic filtering problem consisting of a signal that models the motion of an incompressible fluid below a free surface when the vertical length scale is much smaller than the horizontal one. The evolution of the two-dimensional rotating system is represented by an infinite dimensional stochastic PDE and observed via a finite dimensional observation process. The deterministic part of the SPDE consists of a classical shallow water equation (with an added viscosity term) and a new type of noise, namely the one introduced in [2]. Although this is a single layer model, therefore it does not completely reflect the complex stratification of the real atmosphere, it allows for important geophysical phenomena such as gravity and Rossby waves, eddy formation and geophysical turbulence.

References:

[1] A. Bain, D. Crisan, Fundamentals of Stochastic Filtering, Springer, 2009
[2] D. Holm , Variational Principles for Stochastic Fluid Dynamics, 2015
[3] G. K. Vallis, Atmospheric and Oceanic Fluid Dynamics, 2005

## Well-posedness and scattering of the Chern-Simons-Schrödinger system.

*Lim, Zhuo Min Harold (Cambridge)*

The Chern-Simons-Schrödinger system is a gauge-covariant version of the cubic nonlinear Schrödinger equation in two space dimensions. It describes the effective dynamics of a large system of nonrelativistic charged quantum particles in the plane, interacting with each other and also with a self-generated long-range electromagnetic field. I will present my recent work establishing well-posedness in the energy space and scattering for the defocusing system, which describes a repulsive binary interaction. The scattering result is surprising from a physical point of view, and reflects subtle cancellations in the long-range electromagnetic interactions.